Λ SINGLE-PARTICLE ENERGY LEVELS IN ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ AND ${}^{15}_{\Lambda}C$

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Abstract

In this research work, the single-particle energy levels of Λ-hypernuclear carbon isotopes, namely ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ was investigated. The folding Λ-nucleus potential based on effective Λ-nucleon interaction which is constructed by Akaishi is used. The calculated binding energies of ${}^{13}_{\Lambda}C$ and ${}^{14}_{\Lambda}C$ by applying folding Λ-nucleus potential are two times larger than the experimental results. In the folding process, only Hartree potential called direct term is considered but Fock term which can give the repulsive effect is neglected. The correction term $\Delta V(r)$ of spin and charge dependence of the effective Λ-N interaction is not taken into account. In addition, we assume that the proton density distribution and neutron density distribution are the same. Moreover, the Λ single-particle energy levels of these hypernuclei have also been calculated by using Woods-Saxon potential including spin-orbit interaction. In this calculation, Λ single particle energy levels of s-state and p-state are -11.57 MeV, -1.32 MeV for ${}^{13}_{\Lambda}C$, -12.10 MeV, -1.81 MeV for ${}^{14}_{\Lambda}C$ respectively. The Λ single-particle energy levels of s-state for ${}^{13}_{\Lambda}C$ and ${}^{14}_{\Lambda}C$ are in good agreement with the experimental and theoretical results. The Λ single-particle energy levels of ${}^{15}_{\Lambda}C$ are estimated to be -12.59 MeV for s-state and -2.29 MeV for p-state.

Key words: effective Λ-nucleon interaction, Woods-Saxon potential, Λ single-particle energy, Λ-hypernuclei

Introduction

Hypernuclear physics is an interesting subject and the study of short-lived hypernuclei can provide new information and dynamical features such as hyperon-nucleon interaction and hyperon-hyperon interaction.These interactions are indispensable for the understanding of high-density nuclear

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matter inside neutron stars where hyperons are possibly mixed and playing extremely important roles. Generally, the scattering experiments are the most suitable experiments to provide the determination of any interaction. However, in hypernuclear physics, hyperon–nucleon (YN) scattering experiment are quite difficult to carry out as in nucleon-nucleon (NN) case due to the short life of hyperons. It is impossible to create either the incident hyperon beams or hyperon targets. Therefore, the information of Y-N interaction could be deduced only from the existing data of hypernuclei. As the experimental motivation, the strangeness hypernuclei could be produced from the emulsion experiments and counter experiments (Danysz, M. et al., (1953), Prowse, D.D.J., (1966), Milner, C. et al., (1985), Dluzewski, P. et al., (1988), Hashimoto, A. et al., (1989), Chrien, R.E. et al., (1989) & Dover, C.B. et al., (1989)). The experimental and theoretical investigations of hypernuclear physics, the understanding of the hyperon-nucleon interaction and its role in few-body systems, were made the summary reports (Bando, H. et al., (1990) & Gibson, B.F. and Hungerford III, E.V., (1995)). However, there was still a problem to understand hyperon-nucleon interaction and hyperon-hyperon interaction deeply.

Thus, a new spectroscopic study of ${}^{12}_{\Lambda}C$ by the (π^+, K^+) reaction was reported by Haseagawa (Hasegawa, T. et al., (1996)). The information of some p-shell Λ-hypernuclei observed from (π^+, K^+) reaction with the use of Ge detector array Hyperball was interpreted by employing shell-model calculations. In this calculation, both Λ and Σ channels are included (Millener, D.J., (2008)). From theoretical investigation of the neutron-rich Λ-hypernuclei, new information of hypernuclear physics such as a strong attractive mechanism due to coherent Λ-Σ coupling for single Λ-hypernuclei and the significance of ΛΛ-ΞN coulping effect in formation of double strangeness Λ-hypernuclei could be explained (Akaishi, Y. et al., (2000) & Myint, K.S., et al., (2003)). Moreover, from the theoretical point of view, the structure analysis of Λ-hypernuclei is investigated by applying various models such as a single-particle model, shell model and cluster model. The binding energies of p-shell Λ-hypernuclei such as ${}^{6}_{\Lambda}He, {}^{7,8,9}_{\Lambda}Li$ and ${}^{12,13,14}_{\Lambda}C$ were calculated by using folding potential (Kolesnikov, N.N. and Kalachev, S.A., (2006)).

The structure calculation of ${}^{9}_{\Lambda}Be$, ${}^{13}_{\Lambda}C$ and ${}^{20,21}_{\Lambda}Ne$ were investigated by Kimura. Their calculated ground state binding energy of ${}^{13}_{\Lambda}C$ with the use of an effective ΛN interaction is 11.6 MeV (Kimura, M. et al., (2011)). Thus, we also would like to investigate the structure analysis of Λ-hypernuclei namely, ${}^{13}_{\Lambda}C$ and, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$.

Mathematical Formulation

Derivation of Energy Matrix Elements

In order to calculate the structure analysis of Λ-hypernucleus, we will start the radial part time-independent Schrödinger equation for two-body system as follows.

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right]u(r)=Eu(r). \qquad (1)$$

In this equation, μ is the reduced mass of Λ hyperon and core nucleus. For our calculation, Gaussian basis wave function with expansion coefficients (c_j) and range parameters (b_j) is used,

$$u(r)=r^{(\ell+1)}\sum_j c_j e^{-(r/b_j)^2} \qquad (2)$$

The Schrödinger equation becomes as,

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2} = E\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}. \qquad (3)$$

The above equation was multiplied both sides of the equation by $r^{(\ell+1)}e^{-(r/b_i)^2}$ from the left and integrated through the equation;

$$\int r^{(\ell+1)}e^{-(r/b_i)^2}\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}dr$$

$$=E\int r^{(\ell+1)}e^{-(r/b_i)^2}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}dr. \qquad (4)$$

We, therefore, can define the above equation as

$$\sum_j H_{ij} c_j = E \sum_j N_{ij} c_j \,,$$

where, $$H_{ij} = T_{ij}^{\ell} + F_{ij}^{\ell} + V_{ij}^{\ell} \,.$$

In this equation, T_{ij}^{ℓ} and F_{ij}^{ℓ} are the kinetic energy, centrifugal potential energy matrix elements and V_{ij}^{ℓ} is potential energy matrix elements which are described as follows:

$$T_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} \left\{ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} \right\} r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{5}$$

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} \int r^{(\ell+1)} e^{-(r/b_i)^2} \frac{\ell(\ell+1)}{r^2} r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{6}$$

$$V_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} V(r)\, r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{7}$$

N_{ij}^{ℓ} stands for the norm matrix element,

$$N_{ij}^{\ell} = \int r^{2(\ell+1)} e^{-(r/b_i)^2} e^{-(r/b_j)^2} dr \,. \tag{8}$$

The N_{ij}^{ℓ}, T_{ij}^{ℓ} and F_{ij}^{ℓ} are analytically solved by using standard integral $\int_0^{\infty} x^{2n} e^{-a^2x^2} dx = \frac{(2n-1)!!}{2^{(n+1)}} \frac{\sqrt{\pi}}{a^{(2n+1)}}$, and then the norm matrix element, kinetic energy matrix element, centrifugal potential energy matrix element can be expressed as follows.

$$N_{ij}^{\ell} = \frac{(2\ell+1)!!}{2^{(\ell+2)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2} \right)^{(\ell+\frac{3}{2})}} \tag{9}$$

$$T_{ij}^{\ell} = -\frac{\hbar^2}{2\mu} \left(A - B + C \right) \tag{10}$$

where $$A = \frac{4}{b_j^{\,4}} \frac{(2\ell+3)!!}{2^{(\ell+3)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^{\,2}} + \frac{1}{b_j^{\,2}}\right)^{(\ell+\frac{5}{2})}}$$

$$B = \frac{(4\ell+6)}{b_j^{\,2}} \frac{(2\ell+1)!!}{2^{(\ell+2)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^{\,2}} + \frac{1}{b_j^{\,2}}\right)^{(\ell+\frac{3}{2})}}$$

$$C = \ell(\ell+1) \frac{(2\ell-1)!!}{2^{(\ell+1)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^{\,2}} + \frac{1}{b_j^{\,2}}\right)^{(\ell+\frac{1}{2})}} .$$

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} \ell(\ell+1) \left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{(\ell+1)} \left(\frac{1}{b_i^{\,2}} + \frac{1}{b_j^{\,2}}\right)^{(\ell+\frac{1}{2})}} \right) \qquad (11)$$

The combination of the kinetic energy matrix element and the centrifugal potential matrix element can be written as,

$$T_{ij}^{\ell} + F_{ij}^{\ell} = \frac{\hbar^2}{2M} N_{ij}^{\ell} \frac{(4\ell+6)}{b_i^{\,2} + b_j^{\,2}} . \qquad (12)$$

The potential energy matrix element is

$$V_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} V(r) r^{(\ell+1)} e^{-(r/b_j)^2} dr . \qquad (13)$$

In order to solve the above equation, it is necessary to know the Λ-core nucleus interaction $V(r)$.

Interaction between Λ hyperon and Core Nucleus

For the calculation of potential energy matrix element, two types of potential namely folding potential which is based on effective Λ-N interaction and phenomenological Woods-Saxon Λ-core nucleus potential are used.

(a) Folding potential

The effective ΛN interaction derived by Akaishi (Akaishi.Y., Private Communication) will be firstly used. The Λ-nucleon interaction depends on states such as even state, odd state, spin singlet state and spin triplet state and thus this interaction is expressed in five-range Gaussian form as follows;

$$V_{\Lambda N}=\frac{1}{4}V_{\Lambda N}(^1S_0)+\frac{3}{4}V_{\Lambda N}(^3S_1)\ ,$$

where,
$$V_{\Lambda N}(\vec{R}-\vec{r})=\sum_{k=1}^{5}V_k(k)\exp\left[-\left(\frac{\vec{R}-\vec{r}}{\mu_k}\right)^2\right] \qquad (14)$$

The strength parameters of effective Λ-N interaction in even state and odd state are given in table (1).

Table 1. The strength parameters of effective Λ-nucleon interaction

Even-state		Odd-state	
$V_{\Lambda N-\Lambda N}(^1S_0)$ (MeV)	$V_{\Lambda N-\Lambda N}(^3S_1)$ (MeV)	$V_{\Lambda N-\Lambda N}(^1S_0)$ (MeV)	$V_{\Lambda N-\Lambda N}(^3S_1)$ (MeV)
47.99645	-46.25826	23.462	230.12
-272.7777	-43.83829	-257.22	-614.76
679.7185	493.1045	33.823	855.15
-160.1574	-136.9770	-57.307	-66.964
-2.274696	-0.568783	-0.19762	1.7357

The schematic diagram of a relation between Λ hyperon and core nucleus is shown in Fig. 1.

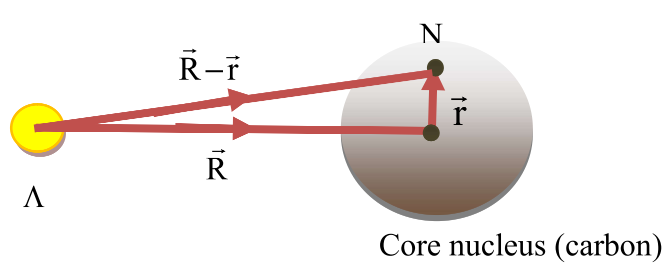

Figure 1. The schematic diagram of a relation between Λ and core nucleus

And thus the Λ-core nucleus interaction is obtained by folding the effective ΛN interaction with the density distribution $\rho(r)$ of core nucleus. Therefore, the effective interaction between Λ and the core nucleus can be written as

$$V_{\Lambda-\text{core}}(R) = \int V_{\Lambda N}(R-r)\,\rho(r)\,d(r)\,. \tag{15}$$

For the density distribution, we will use the proton density or charge density distribution in harmonic oscillator model and we will assume that the proton density distribution is the same as the neutron density distribution. Thus, the density distribution $\rho(r)$ (Dejager, C.W. et al., (1974)) for core nucleus is

$$\rho(r) = \rho_0\left(1+\frac{\alpha}{a^2}r^2\right)e^{-(r/a)^2}. \tag{16}$$

The values of α and a are 1.067 fm, 1.687 fm for ${}^{13}_{\Lambda}C$, 1.403 fm, 1.635 fm for ${}^{14}_{\Lambda}C$ and 1.38 fm, 1.73 fm for ${}^{15}_{\Lambda}C$ respectively. By substituting the density distribution $\rho(r)$, effective interaction in equation (13) and solving it, the Λ-core nucleus interaction can be obtained as

$$V_{\Lambda-\text{core}}(R) = \sum_{k=1}^{5} V_k(k) \quad \rho_0\, e^{-\left(\frac{R}{\mu_k}\right)^2}\left(\frac{\pi}{\frac{1}{\mu_K{}^2}+\frac{1}{a^2}}\right)^{\frac{3}{2}} \exp\left(\frac{R^2}{\mu_k{}^2+\frac{\mu_k{}^4}{a^2}}\right)$$

$$\left[1+\frac{\alpha}{a^2}\left\{\frac{3}{2\left(\frac{1}{\mu_k^{\,2}}+\frac{1}{a^2}\right)}+\frac{R^2}{\mu_k^{\,4}\left(\frac{1}{\mu_k^{\,2}}+\frac{1}{a^2}\right)^2}\right\}\right] \qquad (17)$$

The above equation (17) is the folding potential or Λ-core nucleus interaction for the interested system. After getting the Λ-core nucleus interaction, the potential energy matrix element can be computed by using equation (13) and the result is as follows.

$$V_{ij}^{\ell}=\sum_{k}^{5}V_k\rho_0\left(\frac{\pi}{\frac{1}{\mu^2{}_k}+\frac{1}{a^2}}\right)^{\frac{3}{2}}\frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\frac{1}{\mu_k^2}-\frac{1}{\left(\mu_k^2+\frac{\mu_k^4}{a^2}\right)}\right)^{\ell+\frac{3}{2}}}Q \qquad (18)$$

where $Q=\left(1+\frac{\alpha}{a^2}\left\{1+\frac{3}{2\left(\frac{1}{\mu_k^2}+\frac{1}{a^2}\right)}+\frac{1}{\mu_k^4\left(\frac{1}{\mu_k^2}+\frac{1}{a^2}\right)^2}\frac{(2\ell+3)}{2\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\frac{1}{\mu_k^2}-\frac{1}{\left(\mu_k^2+\frac{\mu_k^4}{a^2}\right)}\right)}\right\}\right)$

Thus the folding potential energy matrix element is

$$V_{ij}^{\ell}=\frac{1}{4}V_{ij}^{\ell}\left({}^1S_0\right)+\frac{3}{4}V_{ij}^{\ell}\left({}^3S_1\right) \qquad (19)$$

(a) Woods-Saxon potential

For this potential, a Λ hyperon moves independently in an average potential well generated by the other nucleons. The phenomenological

Woods-Saxon potential (Dudek, J. et al., (1981)) having the interaction strength $V_0 = 30$ MeV and the nuclear density $\rho(r)$ is

$$V_{w\text{-}s}(r) = -V_0\rho(r) \tag{20}$$

where, $\rho(r) = \dfrac{1}{1+e^{\frac{r-R}{a}}}$, the nuclear radius $R = r_0 A^{\frac{1}{3}} = 1.1 A^{\frac{1}{3}}$ fm and the diffuseness parameter a=0.6 fm. The mass number A is for the core nucleus. We will also consider spin-orbit interaction which is mentioned as followed;

$$V_{\ell s}(r) = V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 (\boldsymbol{\ell}.\mathbf{s})\frac{1}{r}\frac{d\rho(r)}{dr} \text{ and}$$

Thus the potential becomes;

$$V(r) = -V_0\rho(r) + V_{SO}\left(\frac{\hbar}{m_\pi c}\right)^2 (\boldsymbol{\ell}.\mathbf{s})\frac{1}{r}\frac{d\rho(r)}{dr}. \tag{21}$$

In this equation, $\dfrac{\hbar}{m_\pi c}$ is Compton wavelength and spin-orbit constant V_{so} is chosen as 4 MeV. The scalar product of LS coupling is

$$\boldsymbol{\ell}.\mathbf{s} = \frac{1}{2}\left(j(j+1) - \ell(\ell+1) - \frac{3}{4}\right).$$

There are two possible states, $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$, which represents for stretch case and Jackknife case. For stretch case, $j = \ell + \frac{1}{2}$ state, the potential is

$$V(r) = \frac{-V_0}{1+e^{(r-R)/a}} - V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \left(\frac{1}{2}\ell\right)\left(\frac{1}{r}\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\frac{1}{a}\right) \text{ and}$$

For Jackknife case, $j = \ell - \frac{1}{2}$, the potential becomes as

$$V(r)=\frac{-V_0}{1+e^{(r-R)/a}}+V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2\left(\frac{1}{2}(\ell+1)\right)\left(\frac{1}{r}\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\ \frac{1}{a}\right).$$

By using the above two equations, we can calculate the potential energy matrix elements which are described as follows.

$$V_{ij}^{\ell}=\int r^{(2\ell+2)}e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2}\left[\frac{-V_0}{1+e^{(r-R)/a}}+V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2\left(\frac{1}{2}\ell\right)\left(-\frac{1}{r}\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\frac{1}{a}\right)\right]dr \qquad (22)$$

$$V_{ij}^{\ell}=\int r^{(2\ell+2)}e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2}\left[\frac{-V_0}{1+e^{(r-R)/a}}+V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2\left(\frac{-(\ell+1)}{2}\right)\left(-\frac{1}{r}\frac{e^{(r-R)/a}}{(1+e^{(r-R)/a})^2}\frac{1}{a}\right)\right]dr \qquad (23)$$

Equations (17), (22) and (23) are the potential energy matrix elements for the two interaction types. Although the folding potential energy matrix element can be computed analytically, the phenomenological Woods-Saxon potential energy matrix elements cannot be find out the solution. Thus, the latter potential energy matrix element have been solved numerically. In order to calculate the binding energy of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ and Λ-single particle energy levels, power inverse iteration method is used as the numerical calculation for solving nonrelativistic Schrödinger equation.

Results and Discussions

We have calculated Λ single-particle energy in carbon isotopes, ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$, by solving Schrödinger equation with the use of Gaussian basis wave function. Two types of potential namely folding potential and phenomenological Woods-Saxon central potential including spin-orbit coupling are applied in this calculation. The effective Λ-N interaction and phenomenological Woods-Saxon potential which are shown in Fig. 2 have been plotted in order to understand the behavior of the interaction. According to this figure, the effective Λ-N interaction strength for singlet state is appreciably stronger than that in triplet state.

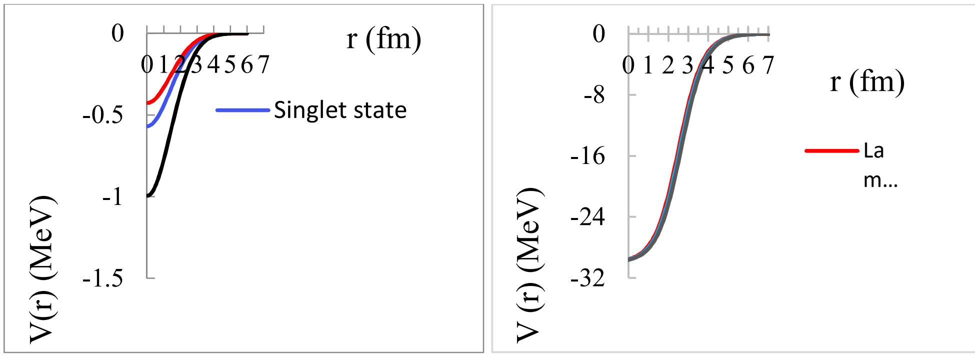

Figure 2. The two potential types; state-dependent effective Λ-N interaction and phenomenological Woods-Saxon potential

In addition, we have also investigated the density distribution of the core nuclei, carbon isotopes, ^{12}C, ^{13}C and ^{14}C which is displayed in Fig. 3. From this graph, we can clearly see that the density distribution of ^{12}C is smoother than that of ^{13}C and ^{14}C.

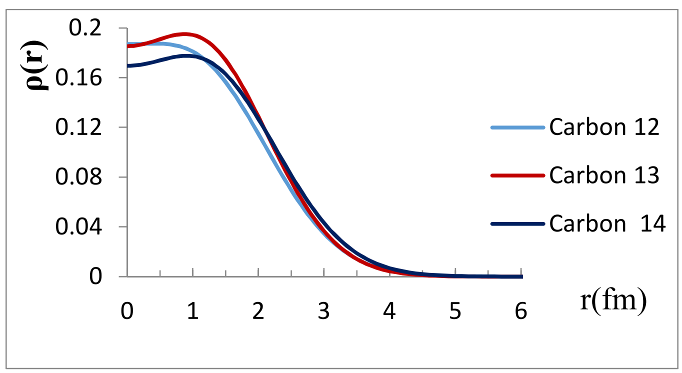

Figure 3. Density distribution of carbon isotope

The Λ single-particle energy levels of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ are investigated by using folding potential based on effective Λ-N interaction. The calculated results are shown in Table (2). The calculated results for ${}^{13}_{\Lambda}C$ and ${}^{14}_{\Lambda}C$ are two times larger than the experimental results (Ajimura, S. et al., (2001) & Kohri, H. et al., (2002)). This is due to the fact that only Hartree term (or) direct term is considered but Fock term or exchange term which can give the repulsive effect is neglected. In addition, the correction term of spin and charge dependence of Λ-N interaction ΔV(**r**) is not taken into account in our calculation. We used the proton density distribution or charge density distribution in harmonic oscillator model and we assume that this proton density distribution is also the same as the neutron density distribution for our system, ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$.

Table 2. The Λ single-particle energy levels of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ by using folding potential

Λ single-particle energy state (MeV)	${}^{13}_{\Lambda}C$	${}^{14}_{\Lambda}C$	${}^{15}_{\Lambda}C$
s-state	-23.89	-27.68	-27.61
Experimental result (s-state): (Ajimura, S. et al., (2001) & Kohri, H. et al., (2002))	-11.69 ± 0.12	-12.17 ± 0.33	-

Moreover, the Λ single-particle energy levels of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ by using phenomenological Woods-Saxon central potential including spin-orbit coupling are displayed in Table (3). By using Woods-Saxon potential including spin-orbit interaction, the Λ single-particle energy levels of s-state and p-state are -11.57 MeV, -1.32 MeV for ${}^{13}_{\Lambda}C$, -12.10 MeV, -1.81 MeV for ${}^{14}_{\Lambda}C$ and -12.59 MeV, -2.29 MeV for ${}^{15}_{\Lambda}C$, respectively. The binding energy of single Λ-hypernucleus, ${}^{13}_{\Lambda}C$ was also investigated by Kimura and his group by using an effective ΛN interaction. Their calculated result is 11.6 MeV (Kimura, M. et al., (2011)). Our calculated results by using Woods-Saxon central potential including spin-orbit interaction are in good agreement with Kimura's result and the experimental results (Ajimura, S. et al., (2001) & Kohri, H. et al., (2002)).

Table 3. The Λ single-particle energy levels of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ by using Woods-Saxon potential including spin-orbit interaction

Λ Single-Particle Energy States (MeV)	${}^{13}_{\Lambda}C$	${}^{14}_{\Lambda}C$	${}^{15}_{\Lambda}C$
s-state	-11.57	-12.10	-12.59
$p_{\frac{3}{2}}$ -state	-1.32	-1.81	-2.29
Experimental result (s-state) (Ajimura, S. et al., (2001) & Kohri, H. et al., (2002))	-11.69 ± 0.12	-12.17 ± 0.33	-

Conclusion

The Λ single-particle energy levels of ${}^{13}_{\Lambda}C$, ${}^{14}_{\Lambda}C$ and ${}^{15}_{\Lambda}C$ have been investigated by solving time-independent Schrödinger equation. Gaussian basis wave function is used for our consideration systems. The folding Λ-nucleus potential based on effective Λ-nucleon interaction and Woods-Saxon potential including spin-orbit interaction are used. The calculated binding energy of ${}^{13}_{\Lambda}C$ and ${}^{14}_{\Lambda}C$ by applying folding Λ-nucleus potential are two times larger than the experimental results. In this folding potential, only Hartree potential or direct term which can give the attractive effect is considered but Fock potential which can give the repulsive effect is neglected. The correction term $\Delta V(r)$ of spin and charge dependence of Λ-N interaction is not taken into account. Moreover, we also assume that the proton density distribution and neutron density distribution are the same. By using Woods-Saxon potential including spin-orbit interaction, the Λ single-particle energy levels of s-state and p-state are -11.57 MeV, -1.32 MeV for ${}^{13}_{\Lambda}C$, -12.10 MeV, -1.81 MeV for ${}^{14}_{\Lambda}C$ and -12.59 MeV, -2.29 MeV for ${}^{15}_{\Lambda}C$, respectively. The Λ single-particle energy levels of s-state for ${}^{13}_{\Lambda}C$ and ${}^{14}_{\Lambda}C$ are in good agreement with the experimental results.

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